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## CALCULUS.

**395. Proposed by W. W. BURTON, Mercer University, Macon, Ga.**

Into a full conical wine glass whose depth is  $a$  and whose angle at the base is  $2\alpha$  there is carefully dropped a spherical ball of such size as to cause the greatest overflow. Show that the radius of the ball is  $a \sin \alpha / (\sin \alpha + \cos 2\alpha)$ .

From Woods and Bailey's *A Course in Mathematics* (1907), Volume I, page 213.

**396. Proposed by ELBERT H. CLARKE, Purdue University.**

The length of the curve  $y = x^n$  from the origin to the point  $(1, 1)$  is given by the formula

$$l = \int_0^1 \sqrt{1 + n^2 x^{2n-2}} dx.$$

Our geometric intuition would tell us that the limit of this length as  $n$  becomes infinite is 2. Give a strict analytic proof that

$$\lim_{n \rightarrow \infty} \int_0^1 \sqrt{1 + n^2 x^{2n-2}} dx = 2.$$

## MECHANICS.

**315. Proposed by H. S. UHLER, Yale University.**

A solid, homogeneous, right, circular cylinder is allowed to move from rest down a circular cylindrical track which is concave upwards. Find the ratio of the radius of the track to the radius of the cylinder when the time of descent through a finite arc to the bottom is the same for the extreme cases of no slipping and zero friction. Show also that the same relation holds for a sphere descending a cylindrical or spherical surface.

**316. Proposed by C. N. SCHMALL, New York, N. Y.**

A body at rest at a point  $R$  begins to move towards a center of force  $F$ . The distance  $RF = d$ , and the force varies inversely as the distance. Two intermediate points in the path are  $P$  and  $Q$ , such that  $FP = kd$ , and  $FQ = k^n d$ . Show that the body will traverse the distance  $QP$  in a maximum of time if  $k = 1/n^{2/(n-1)}$ .

## SOLUTIONS OF PROBLEMS.

## ALGEBRA.

**426. Proposed by HERBERT N. CARLETON, West Newbury, Mass.**

Find all solutions of the equation

$$x^{\sqrt[n]{x}} = x^x.$$

SOLUTION BY J. A. CAPRON, Notre Dame, Ind.

The equation may be written in the form  $x^{1+(1/x)} = x^x$ , or  $x^{(x+1)/x} - x^x = 0$ . Factoring, we have  $x^x [x^{(x+1-x^2)/x} - 1] = 0$ . This equation is equivalent to the two equations  $x^x = 0$  and  $x^{(x+1-x^2)/x} - 1 = 0$ . The first of these equations is satisfied for the value of  $x = -\infty$ . From the second equation, we have, by taking logarithms, the equation

$$\left( \frac{x+1-x^2}{x} \right) \log x = 0.$$

This equation is equivalent to the three equations  $1/x = 0$ ,  $x+1-x^2 = 0$ , and  $\log x = 0$ . From the first of these equations,  $x = \pm \infty$ ; from the second,  $x = (1 \pm \sqrt{5})/2$ ; and from the third,  $x = 1$ .

By substituting the values of  $x$  found above in the original equation, we see that 0 and  $\pm \infty$  are to be rejected. We find, however, by inspection that  $x = 1$  is a root.

Hence, the roots are 1 and  $(1 \pm \sqrt{5})/2$ .

Also solved by ALBERT N. NAUER, A. M. HARDING, C. E. GITHENS, V. M. SPUNAR, ELIJAH SWIFT, W. C. EELLS, G. W. HARTWELL, and the PROPOSER.

**430A. Proposed by H. C. FEEMSTER, York College, Neb.**

Solve the equations

$$\sum_{i=1}^n x_i - x_n = k + \frac{n^2 - 3n + 2}{2} d, \quad (1)$$

$$\sum_{i=1}^n x_i - x_{n-1} = k + \frac{n^2 - 3n + 4}{2} d, \quad (2)$$

$$\sum_{i=1}^n x_i - x_{n-2} = k + \frac{n^2 - 3n + 6}{2} d, \quad (3)$$

$$\begin{array}{cccccccc} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ & & & & & & & \vdots \end{array}$$

$$\sum_{i=1}^n x_i - x_1 = k + \frac{n^2 - n}{2} d. \quad (n)$$

SOLUTION BY A. M. HARDING, Univ. of Arkansas.

Add the given equations and obtain

$$(n-1) \sum_{i=1}^n x_i = nk + \frac{n^3 - 3n^2 + n^2 + n}{2} d,$$

or

$$\sum_{i=1}^n x_i = \frac{n}{n-1} \cdot k + \frac{n(n-1)}{2} d.$$

Subtract each of the given equations from this equation and obtain

$$x_n = \frac{k}{n-1} + (n-1)d,$$

$$x_{n-1} = \frac{k}{n-1} + (n-2)d,$$

$$x_{n-2} = \frac{k}{n-1} + (n-3)d,$$

$$\begin{array}{ccccccc} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{array}$$

$$x_2 = \frac{k}{n-1} + d,$$

$$x_1 = \frac{k}{n-1}.$$

Also solved by NATHAN ALTSHILLER, S. A. JOFFE, J. W. CLAWSON, FRANK R. MORRIS, ELBERT H. CLARKE, HORACE OLSON, N. P. PANDYA, and the PROPOSER.

**431. Proposed by ELMER SCHUYLER, Brooklyn, N. Y.**

Form a magic square of 9 cells such that (the integers being all different) the products of the integers in the rows, columns, and diagonals shall be the same and the smallest product possible.